Forking and superstability in tame abstract elementary classes

Sebastien Vasey

Carnegie Mellon University

August 9, 2014
Classification Theory Workshop
National Institute for Mathematical Sciences, Daejeon, Korea

Introduction

- Forking is one of the key notions of modern stability theory.

Introduction

- Forking is one of the key notions of modern stability theory.
- Is there such a notion outside of first-order (e.g. for logics such as $L_{\omega_1,\omega}$)?

Introduction

- Forking is one of the key notions of modern stability theory.
- Is there such a notion outside of first-order (e.g. for logics such as $L_{\omega_1,\omega}$)?
- One way to formalize this question is to use Shelah's definition of a *good λ-frame*.

Introduction

- Forking is one of the key notions of modern stability theory.
- Is there such a notion outside of first-order (e.g. for logics such as $L_{\omega_1,\omega}$)?
- One way to formalize this question is to use Shelah's definition of a *good λ-frame*.
- Roughly, a good λ-frame is a nice class of models of size λ, together with a forking-like notion for types of singletons.

Introduction

- Forking is one of the key notions of modern stability theory.
- Is there such a notion outside of first-order (e.g. for logics such as $L_{\omega_1,\omega}$)?
- One way to formalize this question is to use Shelah's definition of a *good λ-frame*.
- Roughly, a good λ-frame is a nice class of models of size λ, together with a forking-like notion for types of singletons.
- Shelah showed how to build a good frame using GCH-like set-theoretic assumptions and local model-theoretic hypotheses.

Introduction

- Forking is one of the key notions of modern stability theory.
- Is there such a notion outside of first-order (e.g. for logics such as $L_{\omega_1,\omega}$)?
- One way to formalize this question is to use Shelah's definition of a *good λ-frame*.
- Roughly, a good λ-frame is a nice class of models of size λ, together with a forking-like notion for types of singletons.
- Shelah showed how to build a good frame using GCH-like set-theoretic assumptions and local model-theoretic hypotheses.
- We show how to build one in ZFC, paying with more global (but very natural) model-theoretic hypotheses.

Abstract elementary classes

Definition (Shelah)

Let K be a nonempty class of structures of the same similarity type $L(K)$, and let $\leq$ be a partial order on K. $(K, \leq)$ is an *abstract elementary class (AEC)* if it satisfies:

1. K is closed under isomorphism, $\leq$ respects isomorphisms.
2. If $M \leq N$ are in K, then $M \subseteq N$.
3. Coherence: If $M_0 \subseteq M_1 \leq M_2$ are in K and $M_0 \leq M_2$, then $M_0 \leq M_1$.
4. Downward Löwenheim-Skolem axiom: There is a cardinal $\text{LS}(K) \geq |L(K)| + \aleph_0$ such that for any $N \in K$ and $A \subseteq |N|$, there exists $M \leq N$ containing A of size $\leq \text{LS}(K) + |A|$.
5. Chain axioms: If δ is a limit ordinal, $\langle M_i : i < \delta \rangle$ is a $\leq$-increasing chain in K, then $M := \bigcup_{i<\delta} M_i$ is in K, and:
   - 5.1 $M_i \leq M$ for all $i < \delta$.
   - 5.2 If $N \in K$ is such that $M_i \leq N$ for all $i < \delta$, then $M \leq N$.

- Example: For $\psi \in L_{\omega_1,\omega}$, Φ a countable fragment containing ψ, $K := (\mathrm{Mod}(\psi), \prec_\Phi)$ is an AEC with $\mathrm{LS}(K) = \aleph_0$.

- Example: For $\psi \in L_{\omega_1,\omega}$, Φ a countable fragment containing ψ, $K := (\mathrm{Mod}(\psi), \prec_\Phi)$ is an AEC with $\mathrm{LS}(K) = \aleph_0$.
- The main test question in the study of AECs is the categoricity conjecture:

Conjecture (Shelah)

For every κ there exists a cardinal $\mu = \mu(\kappa)$ such that whenever K is an AEC with $\mathrm{LS}(K) = \kappa$ and K is categorical in *some* cardinal above μ, then K is categorical in *all* cardinals above μ.

Simplifying assumptions

Let K be an AEC.

Definition

- $f : M \to N$ is a (K-)*embedding* if $f : M \cong f[M]$ and $f[M] \leq N$. (From now on, every mapping will be assumed to be an embedding).
- K has *no maximal models* if for any $M \in K$ there exists $N \in K$ so that $M < N$ (i.e. $M \leq N$ and $M \neq N$).
- K has *joint embedding* if for any $M_1, M_2 \in K$, there exists $N \in K$ and $f_\ell : M_\ell \to N$, $\ell = 1, 2$.
- K has *amalgamation* if for any $M_0, M_1, M_2 \in K$ with $M_0 \leq M_\ell$, $\ell = 1, 2$, there exists $N \in K$ and $f_\ell : M_\ell \to N$ so that $f_1 \restriction M_0 = f_2 \restriction M_0$.

More simplifying assumptions

- All these notions can be localized to models of a particular size.

More simplifying assumptions

- All these notions can be localized to models of a particular size.
- If we assume they hold globally, we can build a homogeneous *monster model* $\mathfrak{C}$ in which every model of the AEC embeds.

More simplifying assumptions

- All these notions can be localized to models of a particular size.
- If we assume they hold globally, we can build a homogeneous *monster model* $\mathfrak{C}$ in which every model of the AEC embeds.
- Another simplifying assumption is tameness, a weak compactness property that was first isolated by Grossberg and VanDieren to prove an approximation to Shelah's categoricity conjecture.

Definition (Tameness)

Let K be an AEC with amalgamation. K is *μ-tame* if for any $M \in K$ and distinct $p, q \in S(M)$ there exists $M_0 \leq M$ of size $\leq \mu$ such that $p \upharpoonright M_0 \neq q \upharpoonright M_0$.

Two approaches to AECs

Question (The global approach to AECs)

Work in ZFC, but assume a monster model and maybe some tameness. What can we say about the AEC?

Two approaches to AECs

Question (The global approach to AECs)

Work in ZFC, but assume a monster model and maybe some tameness. What can we say about the AEC?

Question (The local approach to AECs)

Make simplifying assumptions in only a few cardinals. When can we transfer them up? Can we build a structure theory cardinal by cardinal?

Two approaches to AECs

Question (The global approach to AECs)

Work in ZFC, but assume a monster model and maybe some tameness. What can we say about the AEC?

Question (The local approach to AECs)

Make simplifying assumptions in only a few cardinals. When can we transfer them up? Can we build a structure theory cardinal by cardinal?

- This is the approach Shelah adopts in his books on classification theory for AECs.
- Many proofs have a set-theoretic flavor and rely on GCH-like principles.
- The key notion there is that of a good λ-frame, a local AEC version of superstability.

Pre-frame

Definition (Shelah)

Let λ be a cardinal. A *pre-λ-frame* is a triple $\mathfrak{s} = (K, \mathop{\smile}\limits^{\vert}, S^{\mathrm{bs}})$, where K is an AEC with $\lambda \geq \mathrm{LS}(K)$, $K_\lambda \neq \emptyset$, for all $M \in K_\lambda$, $S^{\mathrm{bs}}(M)$ is a set of non-algebraic types, and:

1. $\mathop{\smile}\limits^{\vert}$ is a relation on quadruples of the form (M_0, M_1, a, N), where $a \in N$ and $M_0 \leq M_1 \leq N$ are all in K_λ.
2. The following properties hold:
   - 2.1 Invariance: Both $\mathop{\smile}\limits^{\vert}$ and S^{bs} are invariant under isomorphisms.
   - 2.2 Monotonicity: If $a \mathop{\smile}\limits^{\vert}{}_{M_0}^{N} M_1$, $M_0 \leq M_0' \leq M_1' \leq M_1 \leq N' \leq N \leq N''$ with $a \in N'$ and $N'' \in K_\lambda$, then $a \mathop{\smile}\limits^{\vert}{}_{M_0'}^{N'} M_1'$ and $a \mathop{\smile}\limits^{\vert}{}_{M_0'}^{N''} M_1'$.
   - 2.3 Nonforking types are basic: If $a \mathop{\smile}\limits^{\vert}{}_{M}^{N} M$, then $\mathrm{tp}(a/M; N) \in S^{\mathrm{bs}}(M)$.

Good frame

Definition (Shelah)

$\mathfrak{s} = (K, \mathop{\smile\!\!\!\!\!\!\mid}\,, S^{\text{bs}})$ is a good λ-frame if it is a pre-λ-frame and:

- K_λ has amalgamation, joint embedding, and no maximal models.
- Stability: $|S^{\text{bs}}(M)| \leq \|M\|$ for all $M \in K_\lambda$.
- Density of basic types: If $M < N$ are both in K_λ, then there is $a \in N$ such that $\text{tp}(a/M; N) \in S^{\text{bs}}(M)$.
- Full existence: If $p \in S^{\text{bs}}(M)$ and $N \geq M$, then there exists $q \in S^{\text{bs}}(N)$ extending p that does not fork over M.
- Uniqueness: If $p, q \in S^{\text{bs}}(N)$ do not fork over M and $p \upharpoonright M = q \upharpoonright M$, then $p = q$.

- Symmetry: If $a_1 \underset{M_0}{\overset{N}{\downarrow}} M_2$, $a_2 \in M_2$, and $\mathrm{tp}(a_2/M_0; N) \in S^{\mathrm{bs}}(M_0)$, then there is M_1 containing a_1 and $N' \geq N$ such that $a_2 \underset{M_0}{\overset{N'}{\downarrow}} M_1$.
- Local character: If δ is a limit ordinal, $\langle M_i : i \leq \delta \rangle$ is an increasing chain in K_λ with $M_\delta = \bigcup_{i<\delta} M_i$, and $p \in S^{\mathrm{bs}}(M_\delta)$, then there exists $i < \delta$ such that p does not fork over M_i.
- Continuity: If δ is a limit ordinal, $\langle M_i : i \leq \delta \rangle$ is an increasing chain in K_λ with $M_\delta = \bigcup_{i<\delta} M_i$, $p \in S(M_\delta)$ is so that $p \restriction M_i$ does not fork over M_0 for all $i < \delta$, then p does not fork over M_0.

We say a good frame is *type-full* if the basic types are all the nonalgebraic types (in that case the density of basic types becomes trivial).

Existence of good frames

When do such beasts exist?

Existence of good frames

When do such beasts exist?

Fact (Shelah)

Assume $2^\lambda < 2^{\lambda^+} < 2^{\lambda^{++}}$ *and the weak diamond ideal in* λ^+ *is not* λ^{++}*-saturated.*

Let K *be an AEC and let* $\lambda \geq LS(K)$ *be a cardinal. Assume:*

1. K *is categorical in* λ *and* λ^+.
2. $0 < I(\lambda^{++}, K) < \mu_{unif}(\lambda^{++}, 2^{\lambda^+})$

Then K *has a good* λ^+*-frame.*

Existence of good frames

When do such beasts exist?

Fact (Shelah)

Assume $2^\lambda < 2^{\lambda^+} < 2^{\lambda^{++}}$ *and the weak diamond ideal in* λ^+ *is not* λ^{++}*-saturated.*

Let K *be an AEC and let* $\lambda \geq LS(K)$ *be a cardinal. Assume:*

1. K *is categorical in* λ *and* λ^+*.*
2. $0 < I(\lambda^{++}, K) < \mu_{unif}(\lambda^{++}, 2^{\lambda^+})$

Then K *has a good* λ^+*-frame.*

Existence of good frames from global assumptions

In ZFC, we can show:

Existence of good frames from global assumptions

In ZFC, we can show:

Theorem

Let K be an AEC with a monster model. If K is categorical in a "high-enough" successor λ^+, then K has a type-full good λ-frame.

Existence of good frames from global assumptions

In ZFC, we can show:

Theorem

Let K be an AEC with a monster model. If K is categorical in a "high-enough" successor λ^+, then K has a type-full good λ-frame.

Theorem

Let K be an AEC with a monster model. If K is μ-tame and categorical in a cardinal λ with $\mathrm{cf}(\lambda) > \mu$, then K has a type-full good $(\geq \lambda)$-frame.

Existence of good frames from global assumptions

In ZFC, we can show:

Theorem

Let K be an AEC with a monster model. If K is categorical in a "high-enough" successor λ^+, then K has a type-full good λ-frame.

Theorem

Let K be an AEC with a monster model. If K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$, then K has a type-full good $(\geq \lambda)$-frame.

The first theorem actually follows from the second (using some results from Sh:394).

Proof of the main theorem (sketch)

Theorem

Let K be an AEC with a monster model. If K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$, then K has a type-full good $(\geq \lambda)$-frame.

Proof of the main theorem (sketch)

Theorem

Let K be an AEC with a monster model. If K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$, then K has a type-full good $(\geq \lambda)$-frame.

The idea is to start with μ-nonsplitting as a candidate for our nonforking relation, and to refine it until we obtain all the desired properties. Instead of categoricity, we will assume only:

Proof of the main theorem (sketch)

Theorem

Let K be an AEC with a monster model. If K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$, then K has a type-full good $(\geq \lambda)$-frame.

The idea is to start with μ-nonsplitting as a candidate for our nonforking relation, and to refine it until we obtain all the desired properties. Instead of categoricity, we will assume only:

1. K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains (where $M <_{\mu,\omega} N$ iff there exists $\langle M_i \in K_\mu : i \leq \omega \rangle$ $<_{\text{univ}}$-increasing continuous with $M_0 = M$ and $M_\omega = N$).

Proof of the main theorem (sketch)

Theorem

Let K be an AEC with a monster model. If K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$, then K has a type-full good $(\geq \lambda)$-frame.

The idea is to start with μ-nonsplitting as a candidate for our nonforking relation, and to refine it until we obtain all the desired properties. Instead of categoricity, we will assume only:

1. K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains (where $M <_{\mu,\omega} N$ iff there exists $\langle M_i \in K_\mu : i \leq \omega \rangle$ $<_{\text{univ}}$-increasing continuous with $M_0 = M$ and $M_\omega = N$).
2. All models in K_λ are μ^+-saturated.

Proof of the main theorem (sketch)

Theorem

Let K be an AEC with a monster model. If K is μ-tame and categorical in a cardinal λ with $\mathrm{cf}(\lambda) > \mu$, then K has a type-full good $(\geq \lambda)$-frame.

The idea is to start with μ-nonsplitting as a candidate for our nonforking relation, and to refine it until we obtain all the desired properties. Instead of categoricity, we will assume only:

1. K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains (where $M <_{\mu,\omega} N$ iff there exists $\langle M_i \in K_\mu : i \leq \omega \rangle$ $<_{\text{univ}}$-increasing continuous with $M_0 = M$ and $M_\omega = N$).
2. All models in K_λ are μ^+-saturated.

Both follow from the categoricity hypothesis. Even if we do not assume the second, our nonforking notion will still be well-behaved for μ^+-saturated bases.

Proof sketch (continued)

So we assume K is an AEC with a monster model, μ-tameness, and:

- K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains.
- All models in K_λ are μ^+-saturated.

Proof sketch (continued)

So we assume K is an AEC with a monster model, μ-tameness, and:

- K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains.
- All models in K_λ are μ^+-saturated.

1. Say $p \in S(N)$ does not μ-fork over M iff there exists $M_0 <_{\text{univ}} M$ such that p does not μ-split over M_0.

Proof sketch (continued)

So we assume K is an AEC with a monster model, μ-tameness, and:

- K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains.
- All models in K_λ are μ^+-saturated.

1. Say $p \in S(N)$ does not μ-fork over M iff there exists $M_0 <_{\text{univ}} M$ such that p does not μ-split over M_0.
2. This has some weak versions of the properties we want. We call the resulting object a *skeletal μ-frame*.

Proof sketch (continued)

So we assume K is an AEC with a monster model, μ-tameness, and:

- K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains.
- All models in K_λ are μ^+-saturated.

1. Say $p \in S(N)$ does not μ-fork over M iff there exists $M_0 <_{\text{univ}} M$ such that p does not μ-split over M_0.
2. This has some weak versions of the properties we want. We call the resulting object a *skeletal μ-frame*.
3. Extend the definition of μ-forking to models of size $> \mu$. This gives us more space, so using methods of Shelah and Boney we can prove all the properties of a good frame hold in $K_{\geq\lambda}$ (except perhaps symmetry).

Proof sketch (continued)

So we assume K is an AEC with a monster model, μ-tameness, and:

- K is stable in μ and μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains.
- All models in K_λ are μ^+-saturated.

1. Say $p \in S(N)$ does not μ-fork over M iff there exists $M_0 <_{\text{univ}} M$ such that p does not μ-split over M_0.
2. This has some weak versions of the properties we want. We call the resulting object a *skeletal μ-frame*.
3. Extend the definition of μ-forking to models of size $> \mu$. This gives us more space, so using methods of Shelah and Boney we can prove all the properties of a good frame hold in $K_{\geq\lambda}$ (except perhaps symmetry).
4. Symmetry holds as well: If not, we get the order property, and thus unstability.

An explicit description of forking

Unpacking our definition of forking, we get:

An explicit description of forking

Unpacking our definition of forking, we get:

Proposition

For $M \leq N$ both in $K_{\geq \lambda}$, $p \in S(N)$ does not fork over M if and only if there is $M_0 \leq M$ in K_μ such that p does not μ-split over M_0.

An explicit description of forking

Unpacking our definition of forking, we get:

Proposition

For $M \leq N$ both in $K_{\geq \lambda}$, $p \in S(N)$ does not fork over M if and only if there is $M_0 \leq M$ in K_μ such that p does not μ-split over M_0.

Definition (Shelah)

For $M \leq N$ in K, $M \in K_\mu$, $p \in S(N)$ *μ-splits* over M if there exists $N_1, N_2 \in K_\mu$ with $M \leq N_\ell \leq N$, $\ell = 1, 2$, and $h : N_1 \cong_M N_2$ such that $h(p \upharpoonright N_1) \neq p \upharpoonright N_2$.

Some corollaries that do not mention frames

Let K be an AEC with a monster model. Assume K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$.

Theorem

K has a type-full good $(\geq \lambda)$-frame.

Corollary

Some corollaries that do not mention frames

Let K be an AEC with a monster model. Assume K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$.

Theorem

K has a type-full good $(\geq \lambda)$-frame.

Corollary

1. K has a unique limit model in all $\lambda' \geq \lambda$. More precisely, if δ and ρ are limit ordinals, $\langle M_i \in K_{\lambda'} : i \leq \delta \rangle, \langle N_i \in K_{\lambda'} : i \leq \rho \rangle$ are $<_{\text{univ}}$-increasing continuous, then $M_\delta \cong M_\rho$, and if in addition $M_0 = N_0$, then $M_\delta \cong_{M_0} N_\rho$.

Some corollaries that do not mention frames

Let K be an AEC with a monster model. Assume K is μ-tame and categorical in a cardinal λ with $\text{cf}(\lambda) > \mu$.

Theorem

K has a type-full good $(\geq \lambda)$-frame.

Corollary

1. K has a unique limit model in all $\lambda' \geq \lambda$. More precisely, if δ and ρ are limit ordinals, $\langle M_i \in K_{\lambda'} : i \leq \delta \rangle, \langle N_i \in K_{\lambda'} : i \leq \rho \rangle$ are $<_{\text{univ}}$-increasing continuous, then $M_\delta \cong M_\rho$, and if in addition $M_0 = N_0$, then $M_\delta \cong_{M_0} N_\rho$.
2. K is stable in all cardinals.

The stability spectrum in tame AECs

We can be more precise and establish a partial stability spectrum theorem:

The stability spectrum in tame AECs

We can be more precise and establish a partial stability spectrum theorem:

Theorem

Let K be an AEC with a monster model. Assume K is μ-tame and stable in μ. Let κ be the least regular cardinal such that μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains of cofinality $\geq \kappa$. The following hold:

The stability spectrum in tame AECs

We can be more precise and establish a partial stability spectrum theorem:

Theorem

Let K be an AEC with a monster model. Assume K is μ-tame and stable in μ. Let κ be the least regular cardinal such that μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains of cofinality $\geq \kappa$. The following hold:

1. If $\kappa = \aleph_0$, then K is stable in all $\lambda \geq \mu$.

The stability spectrum in tame AECs

We can be more precise and establish a partial stability spectrum theorem:

Theorem

Let K be an AEC with a monster model. Assume K is μ-tame and stable in μ. Let κ be the least regular cardinal such that μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains of cofinality $\geq \kappa$. The following hold:

1. If $\kappa = \aleph_0$, then K is stable in all $\lambda \geq \mu$.
2. If GCH holds, then K is stable in all $\lambda \geq \mu$ such that $\lambda^{<\kappa} = \lambda$.

The stability spectrum in tame AECs

We can be more precise and establish a partial stability spectrum theorem:

Theorem

Let K be an AEC with a monster model. Assume K is μ-tame and stable in μ. Let κ be the least regular cardinal such that μ-nonsplitting has local character for $<_{\mu,\omega}$-increasing chains of cofinality $\geq \kappa$. The following hold:

1. If $\kappa = \aleph_0$, then K is stable in all $\lambda \geq \mu$.
2. If GCH holds, then K is stable in all $\lambda \geq \mu$ such that $\lambda^{<\kappa} = \lambda$.

Remark

The following were already known:

1. (Shelah) $\kappa \leq \mu^+$.
2. (Grossberg-VanDieren) K is stable in all λ such that $\lambda = \lambda^\mu$.
3. (Baldwin-Kueker-VanDieren) K is stable in μ^+.

Future work: A nonforking notion for models

Can we extend our forking notion to longer types? For example, can we get a nonforking notion $M_1 \overset{M_3}{\underset{M_0}{\mathop{\smile\!\!\!\!\!\!|}}} M_2$?

Future work: A nonforking notion for models

Can we extend our forking notion to longer types? For example, can we get a nonforking notion $M_1 \overset{M_3}{\underset{M_0}{\bigcup}} M_2$?

Some known results from each approach:

Future work: A nonforking notion for models

Can we extend our forking notion to longer types? For example, can we get a nonforking notion $M_1 \underset{M_0}{\overset{M_3}{\downfree}} M_2$?

Some known results from each approach:

1. The local approach: Shelah: under some set-theoretic hypotheses, a good frame can be extended to a nonforking notion for models.

Future work: A nonforking notion for models

Can we extend our forking notion to longer types? For example, can we get a nonforking notion $M_1 \overset{M_3}{\underset{M_0}{\downarrow}} M_2$?

Some known results from each approach:

1. The local approach: Shelah: under some set-theoretic hypotheses, a good frame can be extended to a nonforking notion for models.
2. The global approach: Boney-Grossberg: Assume a monster model, a strong version of tameness, a strong version of stability, and the existence and extension properties for coheir. Then coheir is a nonforking notion for models (in a weaker sense than Shelah's).

Future work: A nonforking notion for models

Can we extend our forking notion to longer types? For example, can we get a nonforking notion $M_1 \underset{M_0}{\overset{M_3}{\perp}} M_2$?

Some known results from each approach:

1. The local approach: Shelah: under some set-theoretic hypotheses, a good frame can be extended to a nonforking notion for models.
2. The global approach: Boney-Grossberg: Assume a monster model, a strong version of tameness, a strong version of stability, and the existence and extension properties for coheir. Then coheir is a nonforking notion for models (in a weaker sense than Shelah's).

Question

Let K be an AEC with a monster model that is tame and totally categorical. Does K have a nonforking notion for models?

감사합니다

- For further reference, see:
  Sebastien Vasey, *Forking and superstability in tame AECs*.
- A preprint can be accessed from my webpage:
  `http://math.cmu.edu/~svasey/`
- For a direct link, you can take a picture of the QR code below: